Math 112 final homework. Due 11/28

For the things of this world cannot be made known without a knowledge of mathematics.
– Francis Bacon

(1) Review what we did in class this week.

(2) Do the following exercises from pages 180-186 in the course notes
- 6.16
- 6.19 parts a, b only
- 6.30
- 6.31
- 7.3 part a [note: we will cover this on Wednesday. If you miss wednesday's class, read section 7.2]
- 7.4

(3) Show that $1243 + 1985^2 - 4827 + 4$ does not equal $4839 + 753^3 - 56(81)$ by finding some n so that they are not congruent mod n.

(4) Do exercise 3 on the encryption reading.

(5) Recall $\mathbb{Z} \times \mathbb{Z}$ arithmetic as we did in class on Monday.
a) List all the elements of $U(\mathbb{Z} \times \mathbb{Z})$

b) Write a multiplication table for $U(\mathbb{Z} \times \mathbb{Z}$ and explain why this is a group. What is the order of the group $U(\mathbb{Z} \times \mathbb{Z})$?

c) Is $U(\mathbb{Z} \times \mathbb{Z})$ cyclic? If so, which elements are generators? If not, why?

d) For each of the elements, find the order of that element in the group $U(\mathbb{Z} \times \mathbb{Z})$.

(6) a) Show that $1000 \equiv 1 \pmod 9$. What about 10000000? Explain why it is true that any power of 10 is congruent to 1 mod 9.

b) Find the residue class of these numbers mod 9: 2000, 50, 700000000.

c) By writing a number as a sum of powers of 10 (e.g. $756 = 700 + 50 + 6$), find the residue class of 1995 (mod 9), and then of 2859571003957 (mod 9).

d) Now we're going to do this for negative numbers: Show that -1000 is congruent to -1 (mod 9), and use this and a similar procedure as above to show that -2345 is congruent to -5. What is the residue class of -5?